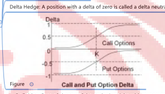
Valuation and Risk Models

Fixed Income Valuation

- Bond Basics
  - Spot Rate
  - Forward Rate
  - Discount Factor
  - Yield to Maturity
- Valution
  - Spot Rate: $Price = \frac{CF_1}{1+s_1} + \ldots + \frac{CF_T}{(1+s_T)^T}$
  - Forward Rate: $Price = \frac{CF_1}{1+f_{0,1}} + \ldots + \frac{CF_T}{(1+f_{0,1})(1+f_{1,2})\ldots(1+f_{T-1,T})}$
  - Discount Factor: $Price = CF_1 \times d(1) + \ldots + CF_T \times d(T)$
  - Bond Replication：Absent confounding factors, identical sets of cash flows should sell for the same price
- Duration: $P = P_0 - D^* P_0 \Delta y$
- Convexity: $P = P_0 - D^* P_0 \Delta y + \frac{1}{2} C P_0 (\Delta y)^2$

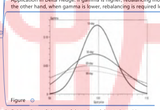
Option Valuation

- Binomial Trees
  - Principles
    - Risk-Neutral Valuation：All cash flows should be discounted to present using the risk-free interest rate
    - Assumption: 1. Stock price follows geometric Brownian motion; 2. $u = 1/d$
  - One-Step Calculation
    - $u = e^{\sigma\sqrt{\Delta t}}, d = e^{-\sigma\sqrt{\Delta t}}$
    - $p(\text{期望值}) = \frac{e^{r\Delta t} - d}{u - d}$ or $p = \frac{e^{(r-q)\Delta t} - d}{u - d}$
    - $\text{Option}(\text{现值}) = [pf_u + (1-p)f_d]e^{-r\Delta t}$
  - Two-Step Calculation: $f = e^{-2r\Delta t}[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd}]$
  - American Options: Make sure that the option value at each node is no less than the intrinsic value
- The Black-Shocley-Merton Model
  - Key Assumptions
    - The price of the underlying asset follows a lognormal distribution
    - The risk-free rate is known and constant
    - The volatility of the underlying asset is known and constant
    - The option valued are European Options
  - Calculation
    - Call: $c = SN(d_1) - Ke^{-rT}N(d_2)$ or $c = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$
    - Put: $p = Ke^{-rT}N(-d_2) = SN(-d_1)$ or $p = Ke^{-rT}N(-d_2) = Se^{-qT}N(-d_1)$
    - $d_{1,2} = \frac{\ln\left(\frac{S}{Ke^{-rT}}\right)}{\sigma\sqrt{T}} \pm \frac{1}{2}\sigma\sqrt{T}$ or $d_{1,2} = \frac{\ln\left(\frac{Se^{-qT}}{Ke^{-rT}}\right)}{\sigma\sqrt{T}} \pm \frac{\sigma\sqrt{T}}{2}$
    - American call: $D_0 > S(1 - e^{-rD_1^{-1}D_1^{-1}})$
    - American put: $D_0 < S(1 - e^{-rD_1^{-1}D_1^{-1}})$
- The Greek Letters
  - Delta
    - Definition: Ratio of the price change of the option to the price change of the underlying assets
    - 范围：-1~1
    - Long call: >0; Short call: <0; Long put: <0; Short put: >0
    - For at the money call, delta ≈ 0.5
    - Delta Hedge: A position with a delta of zero is called a delta neutral position
    - Figure



Call and Put Option Delta

  - Gamma
    - Definition: Rate of change of the option's delta with respect to the price of the underlying asset
    - Gamma is largest when an option is at the money
    - Long call: >0; Short call: <0; Long put: >0; Short put: <0
    - Application in Delta Hedge: If gamma is higher, rebalancing more frequently is required. On the other hand, when gamma is lower, rebalancing is required less frequently
    - Figure



  - Vega
    - Definition: Rate of change of the value of the option with respect to the volatility of the underlying asset
    - Most sensitive to changes in volatility when they are at the money
  - Theta
    - Definition: Rate of change of the value of the option with respect to the volatility of the underlying asset
    - Short-term at the money option has a greatest negative theta
  - Rho
    - Definition: Sensitivity to the interest rates
    - In the money calls and puts are more sensitive to changes in rates than out-of-the-money options

Market Risk Model

- Coherent Properties
  - Monotonicity: $\tilde{R}_1 \geq \tilde{R}_2$, then $\rho(\tilde{R}_1) \leq \rho(\tilde{R}_2)$
  - Subadditivity: $\rho(\tilde{R}_1 + \tilde{R}_2) \leq \rho(\tilde{R}_1) + \rho(\tilde{R}_2)$
  - Positive Homogeneity: $\beta > 0, \rho(\beta\tilde{R}) = \beta\rho(\tilde{R})$
  - Translation Invariance: $\rho(\tilde{R} + c) = \rho(\tilde{R}) - c$
  - Conditional VaR
- Expected Shortfall
  - Sub-additive
  - Average of the worst 100×(1-α)% of losses
- Value at Risk (VaR)
  - Defetion：VaR is the maximum loss over a target horizon and for a given confidence level
  - Parameters
    - Time horizon：should depend on how quickly portfolio can be unwound
    - Confidence level：depends on objectives. Regulators use 99% for market risk and 99.9% for credit/operational risk
  - Properties
    - Not contain worst conditions/tail loss
    - Not sub-additive
  - Calculation
    - $VaR(X\%) = \Pi(R) = q_{0.X} \approx \mu$
    - $VaR(X\%)_{T=date} = VaR(X\%)_{T=date} \times \sqrt{T}$
- Value at Risk for Derivatives
  - Delta-Normal
    - Linear Approximation
      - VaR(Pf)=|-D*×P|×VaR(dy)
      - VaR(df)=|Δ|×VaR(dS)
  - Delta-Gamma
    - Nonlinear Approximation
      - VaR(Pf) =|-D*×P+VaR(dy)+1/2 (C×P)×VaR(dy)^2 |
      - VaR(df)=|Δ×VaR(dS)+1/2 Γ×VaR(dS)^2 |
  - Full Revaluation: Full re-pricing of the portfolio under the assumption that the underlying risk factor(s) are shocked to experience a loss.
    - Historical/Bootstrap Simulation
    - Monte Carlo Simulation
- Volatility Estimation
  - Fat Tails
    - Cause: Volatility is time-varying
    - Solution: Regime-switching volatility model
  - Historical-Based Approach
    - Parametric Model
    - Nonparametric Model
    - Hybrid Approach
  - Implied Volatility-Based Approach: Uses derivative pricing models such as the Black-Scholes-Merton option pricing model to estimate an implied volatility based on current market data

Credit Risk Model

- Expected Loss (EL)
  - property
    - Expect to lose on average over a predetermined period of time
    - Portfolio EL is linear and additive
  - calculation
    - $AE = EE \times LGD_E$
    - $EL = PD \times EA \times LR$
    - $EL_P = \sum_{i=1}^{n} EL_i$
- Unexpected Loss (UL)
  - property
    - Risk arises from the variation in loss levels – which for credit risk is due to unexpected losses
    - Portfolio UL need consider the effects of diversification
  - calculation
    - $UL = EA \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \sigma_{PD}^2}$
    - $\sigma_{PD}^2 = PD \times (1 - PD)$
    - $UL_P = w_1^2 UL_1^2 + w_2^2 UL_2^2 + 2\rho_{12} w_1 w_2 UL_1 UL_2$
- UL Contribution
  - $ULC_1 = \frac{UL_1^2 + \rho_{12}\frac{UL_2}{UL_1}UL_1}{UL_P}$
  - $UL_P = \sum_{i=1}^{n} ULC_i = ULC_1 + ULC_2$

Operational Risk Model

- Define: 操作风险是由内部程序、人员和系统的不完善或失败，或者由于外部事件造成的风险（包含法律风险，但不包含声誉风险和策略风险）
- 资本金计量
  - Basic Indicator Approach
  - Standardized Approach
  - AMA Approach
- Loss Distribution
  - Loss Frequency Distribution：Poisson Distribution
  - Loss Severity Distribution：lognormal probability distribution
  - Loss Distribution：Monte Carlo Simulation
- Features of Stress Testing
  - Advantage
    - Complements value at risk
    - Simple and intuitive
    - Directly examines the tails
  - Disadvantage
    - Highly subjective
    - Implausible scenarios
    - Difficult to interpret

Country Risk

- Sources of Country Risk
  - Economic Growth Life Cycle
  - Political Risk
  - Legal Risk
  - Economic Structure
- Sovereign Default
  - Causes
    - Foreign Currency Default
    - Local Currency Default
  - Consequences
    - GDP growth decline
    - Sovereign ratings decline and borrowing costs increase
    - Political change
    - Fragile banking systems
  - Sovereign Default Spread
    - Advantage
      - Changes occur in real time
      - Adjust quickly to new information
    - Disadvantage
      - Need for a risk-free security
      - Greater volatility
- Rating Agencies
  - Investment/Speculative Grade Ratingd
    - Highest rating. Extremely strong capacity to meet obligations（AAA）
    - Exhibits adequate protection parameters（BBB）
    - Investment Grade Ratings：BBB以上为投资级
    - Speculative Grade Ratings：BBB以下为投机级
  - Transition Matrices：Plot the frequency of rating migrations over time
  - Rating Method
    - At-the-Point-in-Time：Assesses credit quality over the near term
    - Through-the-Cycle：Incorporate business cycles
  - Ratings Change Impact Bond Prices
    - Downgrades have a negative impact on bond prices
    - Upgrades have a positive impact
    - the relationship is statistically stronger for upgrades than downgrades

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